NATURAL CONVECTION ON A MOVING ISOTHERMAL VERTICAL PLATE WITH CHEMICAL REACTION

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Numerical solution of the transient natural convection flow of an incompressible viscous fluid past an impulsively started semiinfinite isothermal vertical plate with mass diffusion is presented here, taking into account a homogeneous chemical reaction of first order. The concentration profiles are compared with the exact solution and are found to be in good agreement. It is observed that, owing to the presence of the first-order chemical reaction, the velocity increases during generative reaction and decreases in destructive reaction.

1. Introduction. Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single-phase volume reaction. In many chemical engineering processes, a chemical reaction between a foreign mass and the fluid in which the plate is moving occurs. These processes take place in numerous industrial applications, e.g., polymer production, manufacturing of ceramics or glassware and food processing. Bourne and Dixon [1] analyzed the cooling of fibers in the formation process.

Stokes [2] presented an exact solution to the Navier–Stokes equations which is the flow of a viscous incompressible fluid past an impulsively started infinite horizontal plate in its own plane. Following Stokes [2] analysis, Soundalgekar [3] was the first to present an exact solution to the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate with mass transfer. Muthukumaraswamy and Ganesan [4] have analyzed the above problem numerically. Das et al. [5] have studied effects of a homogeneous first-order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. The dimensionless governing equations were solved by the usual Laplace-transform technique.

The present investigation, involving the simultaneous effects of heat and mass transfer, is concerned with a numerical study of transient natural convection flow past an impulsively started semiinfinite vertical plate which is subjected to uniform heat and diffusion of a chemically reactive species. The fluids considered in this study are air and water. The governing equations are solved by an implicit finite-difference scheme of Crank–Nicolson type. In order to check the accuracy of our numerical results, the present study is compared with the available exact solution of Das et al. [5] at a lower time level, and they are found to be in good agreement.

2. The Basic Equation. Here the flow of a viscous incompressible fluid past an impulsively started semiinfinite vertical plate with uniform heat and mass diffusion is considered. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first-order chemical reaction between the diffusing species and the fluid. The *x*-axis is taken along the semiinfinite plate in the vertically upward direction and the *y*-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of

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the same temperature and concentration in a stationary condition. At time $t' > 0$, the plate starts moving impulsively in the vertical direction with constant velocity u_0 against the gravitational field. The temperature and the concentration level near the plate are raised uniformly. It is also assumed that there exists a homogeneous first-order chemical reaction between the fluid and species concentration. Then, under usual Boussinesq approximation, the unsteady flow past the semiinfinite vertical plate is governed by the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t'} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta\,(T' - T'_\infty) + + g\beta^*\,(C' - C'_\infty) + v\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial T'}{\partial t'} + u\frac{\partial T'}{\partial x} + v\frac{\partial T'}{\partial y} = \alpha\frac{\partial^2 T'}{\partial y^2}, \tag{3}$$

$$\frac{\partial C'}{\partial t'} + u\frac{\partial C'}{\partial x} + v\frac{\partial C'}{\partial y} = D\frac{\partial^2 C'}{\partial y^2} - K_l C'. \tag{4}$$

The initial and boundary conditions are

$$t' \le 0:\ \ u = 0,\ \ v = 0,\ \ T' = T'_\infty,\ \ C' = C'_\infty,$$

$$t' > 0:\ \ u = u_0,\ \ v = 0,\ \ T' = T'_w,\ \ C' = C'_w,\ \ \text{at } y = 0,$$

$$u = 0,\ \ T' = T'_\infty,\ \ C' = C'_\infty,\ \ \text{at } x = 0, \tag{5}$$

$$u \to 0,\ \ T' \to T'_\infty,\ \ C' \to C'_\infty,\ \ \text{as } y \to \infty.$$

On introducing the nondimensional quantities

$$X = \frac{xu_0}{v},\ \ Y = \frac{yu_0}{v},\ \ U = \frac{u}{u_0},\ \ V = \frac{v}{u_0},\ \ t = \frac{t'u_0^2}{v},$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ \ \mathrm{Gr} = \frac{vg\beta\,(T'_w - T'_\infty)}{u_0^3},\ \ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ \ \mathrm{Gr}_C = \frac{vg\beta^*\,(C'_w - C'_\infty)}{u_0^3},$$

$$\Pr = \frac{v}{\alpha},\ \ \mathrm{Sc} = \frac{v}{D},\ \ K = \frac{vK_l}{u_0^2}$$

$$X = \frac{xu_0}{v},\ \ Y = \frac{yu_0}{v},\ \ U = \frac{u}{u_0},\ \ V = \frac{v}{u_0},\ \ t = \frac{t'u_0^2}{v},$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ \ \mathrm{Gr} = \frac{vg\beta\,(T'_w - T'_\infty)}{u_0^3},\ \ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ \ \mathrm{Gr}_C = \frac{vg\beta^*\,(C'_w - C'_\infty)}{u_0^3},$$

$$\mathrm{Pr} = \frac{v}{\alpha}, \quad \mathrm{Sc} = \frac{v}{D}, \quad K = \frac{vK_l}{u_0^2}$$

Eqs. (1)–(4) are reduced to the following nondimensional form:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{7}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \mathrm{Gr}\,T + \mathrm{Gr}_C\,C + \frac{\partial^2 U}{\partial Y^2}, \tag{8}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 T}{\partial Y^2}, \tag{9}$$

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} = \frac{1}{\mathrm{Sc}}\frac{\partial^2 C}{\partial Y^2} - KC. \tag{10}$$

The corresponding initial and boundary conditions in nondimensional form are

$$t \leq 0: \quad U = 0, \quad V = 0, \quad T = 0, \quad C = 0,$$

$$t > 0: \quad U = 1, \quad V = 0, \quad T = 1, \quad C = 1 \quad \text{at } Y = 0,$$

$$U = 0, \quad T = 0, \quad C = 0 \quad \text{at } X = 0, \tag{11}$$

$$U \to 0, \quad T \to 0, \quad C \to 0 \quad \text{as } Y \to \infty.$$

3. Numerical Procedure. The unsteady, nonlinear coupled equations (7) to (10) with the condition (11) are solved by employing an implicit finite-difference scheme of Crank–Nicolson type. The region of integration is considered as a rectangle with sides $X_{\max} = 1$ and $Y_{\max} = 16$, where $Y_{\max}$ corresponds to $Y = \infty$, which lies very well outside the momentum, energy, and concentration boundary layers. The maximum of Y was chosen as 16 after some preliminary investigations so that the last two of the boundary conditions (11) are satisfied with in the tolerance limit 10^{-5}. After experimenting with a few sets of mesh sizes, the mesh sizes have been fixed at the level $\Delta X = 0.05$ and $Y = 0.25$ with time step $\Delta t = 0.01$. The local truncation error is $O(\Delta t^2 + \Delta Y^2 + \Delta X^2)$ and it tends to zero as Δt, ΔX, and ΔY tend to zero. Hence the scheme is compatible. The finite-difference scheme is unconditionally stable, as discussed by Muthukumaraswamy and Ganesan [4]. Stability and compatibility ensures convergence.

4. Results and Discussion. Representative numerical results for the uniform heat and mass diffusion will be discussed in this section. In order to ascertain the accuracy of the numerical results, the present study is compared with the available exact solution in the literature. The concentration profiles for $K = 0.2$, $\mathrm{Sc} = 0.7$ and 0.9, $\mathrm{Gr} = 2$, $\mathrm{Gr}_C = 5$, and $\mathrm{Pr} = 0.71$ (corresponding to $\eta = Y/2\sqrt{t}$) are compared with the available exact solution of Das et al. [5] at $t = 0.2$ in Fig. 1 and they are found to be in good agreement.

The mass diffusion equation (10) can be adjusted to meet these circumstances if one takes
(i) $K > 0$ for the destructive reaction and
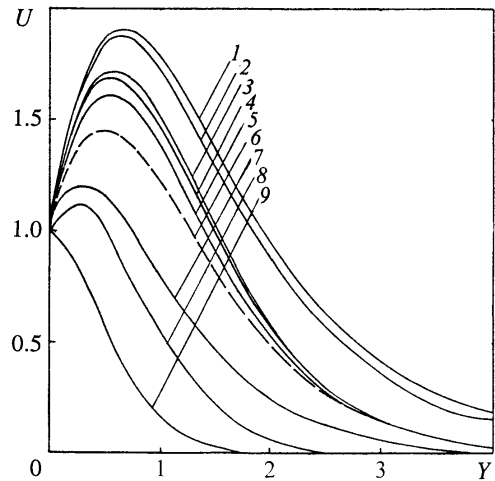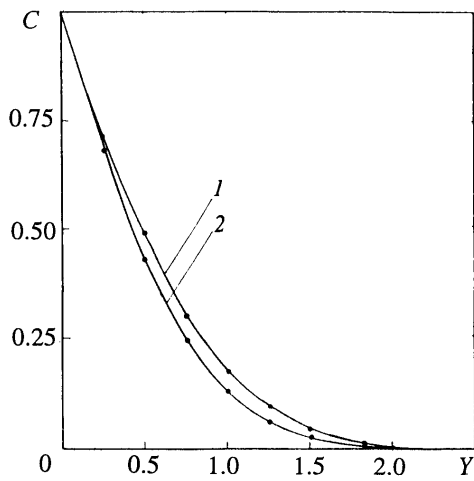(ii) $K < 0$ for the generative reaction.

Fig. 1. Comparison of concentration profiles of the present work (solid lines) with the exact solution of Das et al. (solid circles) for $Gr = 2$, $Gr_C = 5$, $Pr = 0.71$, $K = 0.2$, and $t = 0.2$: 1) $Sc = 0.7$ and 2) 0.9.

Fig. 2. Transient velocity profiles at $X = 1.0$ for $Gr = 2$, $Gr_C = 5$, and $Pr = 0.72$ (solid lines) and 7 (dashed line): 1) $Sc = 0.16$, $K = 0.2$, and $t = 0.73$; 2) 0.16, 0.2, and 4.1^*; 3) 0.6, -2, and 0.79; 4) 0.6, -2, and 7.7^*; 5) 0.6, 0.2, and 10.5^*; 6) 0.6, 0.2, and 12.3^*; 7) 0.16, 0.2, and 0.3; 8) 0.6, -2, and 0.3; 9) 0.6, -2, and 0.13. The asterisk corresponds to steady state.
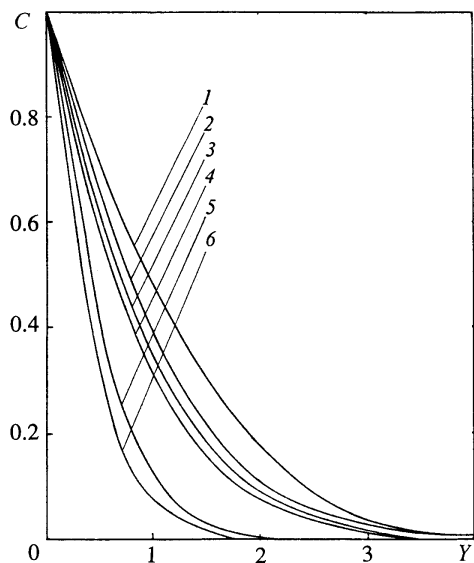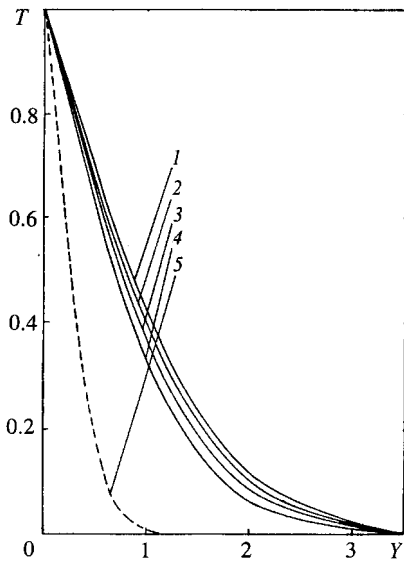

Fig. 3. Steady state temperature profiles at $X = 1.0$ for $Sc = 0.6$ and $Pr = 0.71$ and 7: 1) $Gr = 2$, $Gr_C = 5$, $K = 0.2$, and $t = 10.5$; 2) 2, 5, -2, and 7.7; 3) 5, 5, 0.2, and 9.3; 4) 5, 10, 0.2, and 6.8; 5) 2, 5, 0.2, and 12.3.

Fig. 4. Transient concentration profiles at $X = 1.0$ for $Gr = 2$, $Gr_C = 5$, and $Pr = 0.71$: 1) $Sc = 0.6$, $K = -2$, and $t = 0.57$; 2) 0.6, -2, and 7.7^*; 3) 0.6 -1, and 8.4^*; 4) 0.6, 0.2, and 10.5^*; 6) 0.5, -2, and 0.08. The asterisk corresponds to steady state.
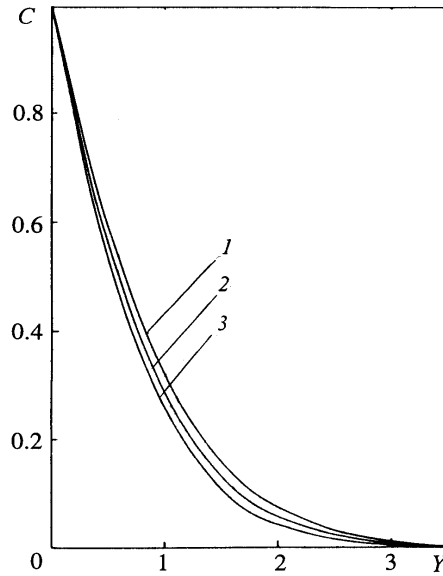
Fig. 5. Steady state concentration profiles at $X = 1.0$ for $\text{Pr} = 0.71$ and $K = 0.2$: 1) $\text{Gr} = 2$, $\text{Gr}_C = 5$, and $t = 10.5$; 2) 5, 5, and 9.3; 3) 5, 10, and 6.8.

The transient velocity profiles for different chemical reaction parameter, Schmidt numbers, and Prandtl number are shown in Fig. 2. The fluids considered in this study are air ($\text{Pr} = 0.71$) and water ($\text{Pr} = 7.0$). The velocity profiles presented are those at $X = 1.0$. It is observed that for $\text{Pr} = 0.71$, $\text{Gr} = 2$, $\text{Gr}_C = 5$, $\text{Sc} = 0.6$ and $K = -2.0$, the velocity increases with time, reaches a temporal maximum around time $t = 0.79$, and becomes steady at time $t = 7.7$. It is observed that the velocity increases during generative reaction and decreases in destructive reaction. It is clear that the velocity increases with decreasing values of the Schmidt number or chemical reaction parameter. The time taken to reach the steady-state increases with increasing Schmidt number or chemical reaction parameter. However, the time required for the velocity to reach steady-state depends upon both the Schmidt number and the chemical reaction parameter. This shows that the contribution of mass diffusion to the buoyancy force increases the maximum velocity significantly. It is observed that the velocity decreases with increasing the Prandtl number.

The transient and steady-state temperature for different values of chemical reaction parameter, thermal Grashof number, mass Grashof number and Prandtl number are shown in Fig. 3. It is observed that the temperature increases with increasing values of the chemical reaction parameter, and decreasing values of the Prandtl number. There is a fall in temperature due to an increasing thermal Grashof number or mass Grashof number. Such a study found useful in identification of the best catalysts.

The effect of chemical reaction parameter and Schmidt number are very important in concentration field. The transient and steady-state concentration profile for different chemical reation parameter are shown in Fig. 4. There is a fall in concentration due to increasing values of the chemical reaction parameter. The concentration profiles for different values of the thermal Grashof number and mass Grashof number are shown in Fig. 5. It is observed that in the presence of a destructive reaction, the concentration increases with decreasing thermal Grashof number or mass Grashof number.

Knowing the concentration field, it is customary to study the rate of concentration in their transient and steady-state conditions. The dimensionless local as well as average Sherwood number are given by the following expressions:

$$\text{Sh}_x = -X \left(\frac{\partial C}{\partial Y} \right)_{Y=0}, \tag{12}$$
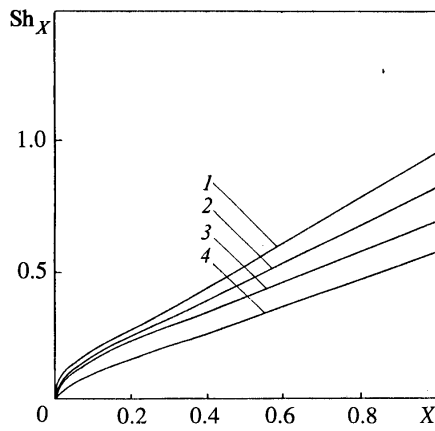
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Fig. 6. Local Sherwood number for 1) $Gr_C = 2$, $Gr = 5$, and $Pr = 0.71$:
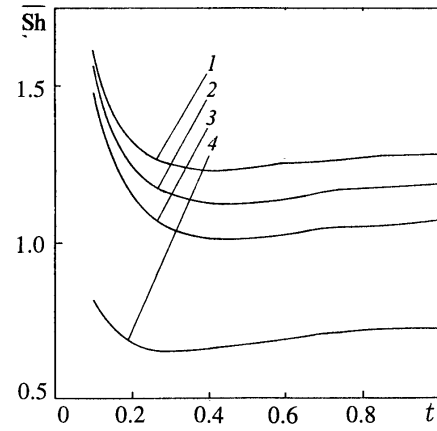1) $Sc = 0.6$ and $K = 0.2$; 2) 0.6 and -1; 3) 0.6 and -2; 4) 0.16 and 0.2.

Fig. 7. Average Sherwood number. For notation, see Fig. 6.

$$\overline{Sh} = -\int_0^1 \left(\frac{\partial C}{\partial Y}\right)_{Y=0} dX. \tag{13}$$

The derivatives involved in Eqs. (12) and (13) are evaluated using five-point approximation formula and then the integrals are evaluated using Newton–Cotes closed integration formula.

The local Sherwood number for different chemical reaction parameters and Schmidt number is shown in Fig. 6. The rate of mass transfer increases with increasing chemical reaction parameter or Sc. It is observed that the rate of concentration increases during destructive reaction and decreases in generative reaction. The effects of Sc and the chemical reaction parameter on the average values of the Sherwood number are shown in Fig. 7. The average Sherwood number increases with increasing chemical reaction parameter or Sc.

Conclusions. A detailed numerical study has been carried out for the flow past an impulsively started semiinfinite vertical plate with uniform heat and diffusion of chemically reactive species. The dimensionless governing equations are solved numerically. The fluids considered in this study are air and water. This study has been compared with the exact solution available in the literature, and they are found to be in good agreement. It is observed that the velocity and concentration increase during generative reaction and decrease in destructive reaction. It is found that the number of time steps to reach steady-state depends strongly on the chemical reaction parameter.

NOTATION

C', concentration; C, dimensionless concentration; D, mass diffusion coefficient; g, acceleration due to gravity; k, thermal conductivity; Gr_C, mass Grashof number; Gr, thermal Grashof number; K, dimensionless chemical reaction parameter; K_l, chemical reaction parameter; Pr, Prandtl number; Sc, Schmidt number; Sh_x, dimensionless local Sherwood number; Sh, dimensionless average Sherwood number; T', temperature; T, dimensionless temperature; t', dimensionless time; u_0, velocity of the plate; u and v, velocity components in the x and y directions, respectively; U and V, dimensionless velocity components in the X and Y directions, respectively; x, spatial coordinate along the plate; X, dimensionless spatial coordinate along the plate; y, spatial coordinate normal to the plate; Y, dimensionless spatial coordinate normal to the plate; α, thermal diffusivity; β, volumetric coefficient of thermal expansion; β^*, volumetric coefficient of expansion with concentration; μ, dynamic viscosity; ν, kinematic viscosity. Subscripts: w, conditions on the wall; ∞, free stream conditions.

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